Uranium Series Disequilibria

Part III Magma Dynamics: Practical 1

Some of the most powerful constraints on melt extraction come from studies of uranium series disequilibria. The observations provide constraints on the residual porosity, the rate of melting, and the rate of melt transport. The aim of this practical is to show you how such constraints arise from some very simple models of mantle melting.

Part of the decay series of ^{238}U is:

$$^{238}\mathrm{U} \to {}^{234}\mathrm{Th} \to {}^{234}\mathrm{Pa} \to {}^{234}\mathrm{U} \to {}^{230}\mathrm{Th} \to {}^{226}\mathrm{Ra}$$

In radioactive equilibrium ^{230}Th is generated at the same rate by the decay of ^{234}U as it decays to ^{226}Ra. The relevant half-lives and partition coefficients are given below.

Quantity	Symbol	Value
Half-life of ^{230}Th	$\tau_{1/2}^{230\mathrm{Th}}$	75,500 years
Half-life of ^{238}U	$\tau_{1/2}^{238\mathrm{U}}$	4.46×10^{9} years
Half-life of ^{226}Ra	$\tau_{1/2}^{226\mathrm{Ra}}$	1,599 years
Partition coefficient of U	D^{U}	1.2×10^{-3}
Partition coefficient of Th	D^{Th}	3.1×10^{-4}
Partition coefficient of Ra	D^{Ra}	10^{-6}

Recall that the decay constant λ is related to the half-life $\tau_{1/2}$ by

$$\lambda = \frac{\ln 2}{\tau_{1/2}}. \tag{1}$$

The decay equation

^{230}Th is produced by the radioactive decay of ^{238}U, a very long-lived isotope. In turn, ^{230}Th undergoes radioactive decay to ^{226}Ra. The evolution of the concentration of ^{230}Th can be described by

$$\frac{\mathrm{d}c^d}{\mathrm{d}t} = -\lambda^d c^d + R \tag{2}$$

where c^d is the concentration of ^{230}Th, λ^d is the decay constant of ^{230}Th, and R is the rate of production of ^{230}Th by ^{238}U. R is essentially a constant because the half-life of ^{238}U is so long.

1. Suppose that all the ^{230}Th is removed from the system at time zero, such that $c^d(0) = 0$. Integrate the ODE (2) to find $c_d(t)$. On what time scale does $c^d(t)$ return to equilibrium and what is its equilibrium value?

Melt fraction

U-series can be used to estimate the melt fraction present in the source. Consider a partially molten rock that is in chemical and radioactive equilibrium. Chemical equilibrium can be described in terms of a constant partition coefficient D,

$$D = \frac{c_s}{c_l}, \tag{3}$$

where c_s represents the concentration of a given element in the solid and c_l represents the concentration of a given element in the liquid. The bulk concentration $\bar{c}$ of an element is given by

$$\bar{c} = \phi c_l + (1-\phi)c_s \tag{4}$$

where ϕ is the porosity (volume fraction of melt present).

The activity a of an isotope is the product of its concentration and its decay constant ($a = \lambda c$). In radioactive equilibrium the total activities of parent (e.g. ^{238}U) and daughter (e.g. ^{230}Th) must be the same, so that

$$\lambda^p \bar{c}^p = \lambda^d \bar{c}^d, \tag{5}$$

where superscripts p refer to the parent isotope and d the daughter.

2. Show that the activity ratio *in the melt*, defined by

$$r \equiv \frac{\lambda^d c_l^d}{\lambda^p c_l^p} \tag{6}$$

   is given by

$$r = \frac{\phi + D^p}{\phi + D^d} \tag{7}$$

   under the assumption $\phi \ll 1$.

3. Typical measured MORB activity ratios are

$$\left(\frac{^{230}\mathrm{Th}}{^{238}\mathrm{U}}\right) = 1.15, \quad \left(\frac{^{226}\mathrm{Ra}}{^{230}\mathrm{Th}}\right) = 1.6.$$

   Use (7) and the table of values to estimate the melt fraction in the source.

Melting rate

A more sophisticated model of U-series takes account of the fact that melt is generated at a certain rate, and that rate might be so fast that radioactive equilibrium is not maintained in the source. A simple model with instantaneous melt extraction yields the activity ratio formula

$$r = \frac{\phi_0 + D^p + \dfrac{1}{\lambda^d}\dfrac{\mathrm{d}F}{\mathrm{d}t}}{\phi_0 + D^d + \dfrac{1}{\lambda^d}\dfrac{\mathrm{d}F}{\mathrm{d}t}} \tag{8}$$

where ϕ_0 is the residual porosity (the volume fraction of melt that remains in chemical equilibrium with the solid), $\mathrm{d}F/\mathrm{d}t$ is the melting rate, and λ^d is the decay constant of the daughter isotope.

4. Estimate the melt productivity with depth ($\mathrm{d}F/\mathrm{d}z$) in m^{-1} for a mid-ocean ridge. An order of magnitude estimate will suffice.

5. Assuming an upwelling rate w of 10 mm yr^{-1}, estimate the melting rate $\mathrm{d}F/\mathrm{d}t$.

6. Suppose a residual porosity of $\phi_0 = 0.04\%$ remains in chemical equilibrium with the residue of melting. Starting from a unmolten state, use your answer to the previous part to estimate the time τ_{melt} it takes to produce this fraction of melt. Would ^{230}Th establish radioactive equilibrium over this time period? Would ^{226}Ra?

7. Produce another estimate of melting rate $\mathrm{d}F/\mathrm{d}t$ using (8) and the MORB activity ratio $\left(^{230}\mathrm{Th}/^{238}\mathrm{U}\right) = 1.15$, assuming a residual porosity of $\phi_0 = 0.04\%$. Is your answer significantly different if you assume a residual porosity of zero? Is your answer significantly different to your answer to 5?

8. Show that with your estimated melting rate from the previous part, and a residual porosity of $\phi_0 = 0.04\%$, you can get a satifactory match to the $\left(^{226}\mathrm{Ra}/^{230}\mathrm{Th}\right) = 1.6$ observation.

Melt transport

Thus far it has been assumed that melt transport to the surface is instantaneous. If disequilibrium signals are to be observed at the surface, the process of melt transport needs to be fast enough that radioactive equilibrium is not re-established in the melt. This is rather difficult to do by diffuse porous flow, as the following exercise demonstrates.

For low porosities, the permeability of a partially molten rock can be described by

$$k = \frac{d^2\phi^2}{C} \tag{9}$$

where C is a numerical constant typically around 1600, d is the grain size, and ϕ is the porosity. Melt transport by diffuse porous flow is described by Darcy's law

$$\phi v_l = \frac{k\Delta\rho g}{\mu} \tag{10}$$

where v_l is the melt velocity, $\Delta\rho$ is the density difference between melt and solid, g is the acceleration due to gravity, and μ is the viscosity of the melt.

9. Assuming the following values, $\Delta\rho = 500$ kg m^{-3}, $g = 10$ m s^{-2}, $\phi = 0.01$, $d = 1$ mm, $\mu = 1$ Pa s, estimate the time it takes for melt to travel to the surface from 60 km depth. If this were the transport time, would we expect to see U-series disequilibrium signals at the surface?

References

This practical is closely based on the following article, which I thoroughly recommend you read:

McKenzie D. (2000) Constraints on melt generation and transport from U-series activity ratios. *Chem. Geol.* **162** 81-94 doi:10.1016/S0009-2541(99)00126-6

There are several more sophisticated models of U-series around, one of which is available as a nice web-app at `http://www.ldeo.columbia.edu/~mspieg/UserCalc/` and is described in:

Spiegelman M. (2000) UserCalc: A Web-based uranium series calculator for magma migration problems. *Geochem. Geophys. Geosyst.* **1** 1999GC000030 doi:10.1029/1999GC000030

Answers

1. $c_d(t) = \dfrac{R}{\lambda^d}\left(1 - \exp\left(-\lambda^d t\right)\right)$. The equilibrium value is $c^d = R/\lambda_d$ and it recovers to its equilibrium value on a characteristic time scale of $1/\lambda^d$.

3. $\phi(^{230}\mathrm{Th}) = 0.6\%$, $\phi(^{226}\mathrm{Ra}) = 0.05\%$.

4. An order of magnitude estimate can be obtained by assuming 20% of the mantle melts over an interval of 60 km, yielding $\mathrm{d}F/\mathrm{d}z = 0.2/(60\,\mathrm{km}) \approx 3 \times 10^{-6}\ \mathrm{m}^{-1}$.

5. $\dfrac{\mathrm{d}F}{\mathrm{d}t} = w\dfrac{\mathrm{d}F}{\mathrm{d}z} \approx 3 \times 10^{-8}\ \mathrm{yr}^{-1}$.

6. $\tau_{\mathrm{melt}} = \dfrac{\phi_0}{\mathrm{d}F/\mathrm{d}t} = 12,000$ years.
   $\tau_{1/2}^{226\mathrm{Ra}} < \tau_{\mathrm{melt}} < \tau_{1/2}^{230\mathrm{Th}}$, so we expect ^{226}Ra to be in radioactive equilibrium, but ^{230}Th not to be. As a consequence ^{230}Th is typically more sensitive to the rate of melting, and ^{226}Ra more sensitive to the residual porosity.

7. $\mathrm{d}F/\mathrm{d}t = 4.8 \times 10^{-8}\ \mathrm{yr}^{-1}$ with $\phi_0 = 0.04\%$. With $\phi_0 = 0$, $\mathrm{d}F/\mathrm{d}t = 5.2 \times 10^{-8}\ \mathrm{yr}^{-1}$, not significantly different (demonstrating that ^{230}Th is not that sensitive to residual porosity in this range). This is also not significantly different from the answer to 5, and could be achieved by a small increase in the upwelling rate.

8. With these parameters, $\left(^{226}\mathrm{Ra}/^{230}\mathrm{Th}\right) = 1.6$ as desired (thanks to the carefully-chosen residual porosity!)

9. $\tau_{\mathrm{transport}} = \frac{D}{v_l} \approx 60,000$ years. You would expect ^{226}Ra to be back in radioactive equilibrium in this time, and to have reduced the ^{230}Th signal somewhat also. ^{226}Ra disequilibrium is observed, so diffuse porous flow seems inconsistent with the U-series observations.