UNIVERSITY OF CAMBRIDGE

Part III Magma Dynamics – Past Practical Exam Answers

1

(a) Sketches should show the following:

$q(z)$: From (4), $q = FV_0$ so q varies linearly from 0 at the bottom of the column to $F_{\max}V_0 = 6$ mm yr^{-1} at the top of the column.

$\phi(z)$: Eliminating q from (2), (3), and (4) yields $\phi = \left(\dfrac{\mu F V_0}{k_0 \Delta\rho g}\right)^{1/2}$, so ϕ varies as $\sqrt{z}$, going from 0 to $\phi_{\max} = 0.006 = 0.6\%$.

$v_f(z)$: From (4) and expression for ϕ it follows that $v_f = \left(\dfrac{k_0 \Delta\rho g F V_0}{\mu}\right)^{1/2}$, so v_f varies as $\sqrt{z}$, going from 0 to $v_{f,\max} = 1$ m yr^{-1}.

$v_s(z)$: From (1) and (4) $v_s = (1 - F)V_0$, so v_s reduces linearly from $V_0 = 30$ mm yr^{-1} at the base of the column to $(1 - F_{\max})V_0 = 24$ mm yr^{-1} at the top.

$\mathcal{P}(z)$: $C = \dfrac{\mathrm{d}v_s}{\mathrm{d}z} = -\dfrac{\mathrm{d}F}{\mathrm{d}z}V_0$. Since $\mathrm{d}F/\mathrm{d}z = F_{\max}/h$ is constant, there is no variation in z. $C = -F_{\max}V_0/h = -10^{-7}$ yr^{-1} $= -3.2 \times 10^{-15}$ s^{-1}. $\mathcal{P} = \zeta C = -45$ kPa (negative sign means medium is compacting).

(b) From (9) the mantle cools by $\Delta T = LF_{\max}/C_p = 98$°C from the bottom to the top of the column.

(c) (i) $\dfrac{\mathrm{d}z}{\mathrm{d}t} = v_{f,\max}\left(\dfrac{z}{h}\right)^{1/2} \implies t = \dfrac{2h}{v_{f,\max}} = 120{,}000$ yr

(i) $\dfrac{\mathrm{d}z}{\mathrm{d}t} = V_0\left(1 - \dfrac{F_{\max}z}{h}\right) \implies t = -\dfrac{\ln(1 - F_{\max})}{F_{\max}}\dfrac{h}{V_0} = 2.2$ Myr

(d) If the upwelling melt suddenly encounters a freezing boundary, the melt will pond there and form a decompaction channel. This will lead to a local increase in porosity in the neighbourhood of the boundary over a lengthscale given by the compaction length. The height of the decompaction channel is

$$\delta = \sqrt{\frac{k_\phi}{\mu}\left(\zeta + \tfrac{4}{3}\eta\right)} = \sqrt{\frac{F_{\max}V_0}{\Delta\rho g}\left(\zeta + \tfrac{4}{3}\eta\right)} = 1 \text{ km}$$

(e) The unloading causes all the material in the column to undergo a decompression of $\Delta P = -\rho_i g t$ where $t = 2$ km is the thickness of the ice. Since the melt productivity is uniform in the column, and can be calculated as $\mathrm{d}F/\mathrm{d}P = -F_{\max}/(\rho_s g h)$, it follows that $F(z)$ increases throughout by an amount $\Delta F = -\rho_i g t\, \mathrm{d}F/\mathrm{d}P = F_{\max}(\rho_i/\rho_s)(t/h) = 0.002 = 0.2\%$.

2

(a) $\dfrac{dF}{dt} = \lambda^d \dfrac{\phi_0(1-r) + D^p - rD^d}{r-1} = \underline{2.2 \times 10^{-7}\ \text{yr}^{-1}}$.

(b) $(^{230}\text{Th}/^{238}\text{U}) = \underline{1.08}$

(c) $v_0 = \dfrac{1}{\rho g} \dfrac{dF/dt}{dF/dP} = \underline{68\ \text{mm yr}^{-1}}$

(d) $Q_B = \underline{1.2\ \text{Mg s}^{-1}}$

(e) Since $v \propto e^{-\rho/a}$, $\left.\dfrac{dF}{dt}\right|_\rho = \left.\dfrac{dF}{dt}\right|_{\text{centre}} e^{-\rho/a}$. Thus melting rates at 150 km are reduced by a factor of $e^{-150/200} = 0.472$ from that at centre (i.e. $dF/dt = 1.0 \times 10^{-7}\ \text{yr}^{-1}$). Thus $(^{231}\text{Pa}/^{235}\text{U}) = \underline{1.42}$, $(^{230}\text{Th}/^{238}\text{U}) = \underline{1.14}$

(f) Integration of the ODE yields $r_0 = 1 + (r-1)\exp\left(\lambda^d t\right)$ where r is the present-day activity ratio, and r_0 is the initial ratio. Substitution yields $(^{231}\text{Pa}/^{235}\text{U})_0 = \underline{1.31}$.

(g) The $\left(^{226}\text{Ra}/^{230}\text{Th}\right)$ activity ratio is sensitive to both the residual porosity and the time scale for melt transport, and could thus provide constraints on these two parameters. However, it is not particularly sensitive to the rate of melting, because the half life of ^{226}Ra is short compared to the melt generation time scale.

3

(a) $\dot{\gamma} = \frac{\gamma}{t} = 2.3 \times 10^{-5}\ \mathrm{s}^{-1}$

(b) $\eta = \frac{\sigma}{\dot{\gamma}} = 2.2 \times 10^{12}$ Pa s

(c) $K_\phi = 2.5 \times 10^{-17}\ \mathrm{m}^2$

(d) $\delta = 4.0$ mm. Compaction length can be defined in many ways e.g. length scale over which ϕ decreases to ϕ/e in a compacting column; length scale over which solid and fluid flow are coupled; or length scale over which melt pressure and porosity gradients can be sustained.

(e) The presence of melt reduces the viscosity by offering a fast path for diffusion. It is diffusion distances that the control the effective viscosity during diffusion creep.

Differentiation yields $\alpha = \frac{1}{\sqrt{\phi\phi_d} - \phi} = 27$

(f) Note that $\nu = 1/3$. Largest growth rate is for melt bands at 45° to the shear plane (when $\sin 2\theta = 1$).

Shorter wavelengths grow faster than longer wavelengths, with all wavelengths less than the compaction length growing at about the same rate. A good answer here might give a sketch of growth rate against wavelength or wavenumber to demonstrate this. $\dot{s}_{\mathrm{max}} = 2\alpha\nu\dot{\gamma} = 4 \times 10^{-4}\ \mathrm{s}^{-1}$. $\dot{s}_{\mathrm{max}} t = 36 \gg 1$ so one expects a significant growth of melt bands (neglecting the fact that during shear, melt bands will rotate out of a favourable orientation for growth).

(g) There are two key things to mention: First the linear stability analysis does not predict a preferred wavelength of the instability: formally the fastest growing wavelength is zero wavelength. Additional physics needs to be introduced to determine the melt band widths and spacings (such as surface tension effects). Secondly, the given theory is for a Newtonian diffusion creep rheology, which always gives 45° melt bands — lab experiments show shallower angles which has been attributed to power-law rheologies or anisotropy in viscous properties.

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